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Non-trivial spacetime effects in a supersymmetric model

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Abstract
We study an $N = 1$ supersymmetric model in an $S^1 \times R^3$ spacetime. We find that by choosing appropriate boundary conditions for the contributing fields supersymmetry can be preserved. However, if we add a hard supersymmetry breaking term, we observe that for small values of the length of the S^1 dimension, supersymmetry remains unbroken and breaks spontaneously when the length exceeds a critical value. The final picture resembles the first-order phase transition picture. A toy cosmological application is discussed.

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Introduction

In general, when studying supersymmetric theories in flat spacetime, the background metric is assumed to be ordinary Minkowski. Spacetime topology may affect the boundary conditions of the fields that are integrated in the path integral. Given a class of metrics, several spacetime topologies are allowed. Here we shall focus on a model that has $S^1 \times R^3$ topology underlying the spacetime, S^1 refers to a spatial dimension. The specific topology is a homogeneous topology of the flat Clifford–Klein type [1]. Non-trivial topology implies non-trivial field configurations that enter dynamically in the action. We shall investigate their impact using the effective potential method [2, 6]. It is known that when supersymmetric theories are studied in non-trivial spacetimes (or at finite temperature), supersymmetry is, in general, spontaneously broken. This is due to the appearance of vacuum terms which have different coefficients for fermions and bosons. As a result, the effective potential of the theory has no longer its minimum at zero, thus supersymmetry is broken. This quite general phenomenon can only be avoided, if in some way these vacuum terms are canceled [12].

The existence of non-trivial field configurations due to non-trivial topology (twisted fields) was first pointed out by Isham [3] and then adopted by others [4, 5, 13]. In our case, the topological properties of $S^1 \times R^3$ are classified by the first Stieffel class $H^1(S^1 \times R^3, Z_{\tilde{2}})$

which is isomorphic to the singular (simplicial) cohomology group $H_1(S^1 \times R^3, Z_2)$ because of the triviality of the $Z_{\tilde{2}}$ sheaf. Now, it is known that $H^1(S^1 \times R^3, Z_{\tilde{2}}) = Z_2$ classifies the twisting of a bundle. To be exact, it describes and classifies the orientability of a bundle globally. In our case, the classification group is Z_2, and we have two locally equivalent bundles, which are however different globally, i.e. cylinder-like and moebius strip-like. The mathematical lying behind is to find the sections that correspond to these two bundles, classified completely by Z_2 [3]. The sections we shall consider are real scalar fields and Majorana spinor fields. These fields carry a topological number called moebiosity (twist), which distinguishes between twisted and untwisted fields. The twisted fields obey anti-periodic boundary conditions, while untwisted fields periodic in the compactified dimension (see below). Normally one takes scalars to obey periodic and fermions anti-periodic boundary conditions, disregarding all other configurations that may arise from non-trivial topology. Here we shall consider all these configurations. If φ_u, φ_t and ψ_t, ψ_u denote the untwisted and twisted scalar and twisted and untwisted spinor respectively, then the boundary conditions in the S^1 dimension are $\varphi_u(x,0) = \varphi_u(x,L)$ and $\varphi_t(x,0) = -\varphi_t(x,L)$ for scalars, and $\psi_u(x,0) = \psi_u(x,L)$, $\psi_t(x,0) = -\psi_t(x,L)$ for fermions, where x stands for the remaining two spatial and one time dimensions not affected by the boundary conditions. Spinors (both Dirac and Majorana) still remain Grassmann quantities. We assign the untwisted fields twist h_0 (the trivial element of Z_2) and the twisted fields twist h_1 (the non-trivial element of Z_2). Recall that $h_0 + h_0 = h_0$ $(0+0=0)$, $h_1 + h_1 = h_0$ $(1+1=0)$, $h_1 + h_0 = h_1$ $(1+0=1)$. We require the Lagrangian to have h_0 moebiosity. The topological charges flowing at the interaction vertices must sum to h_0 under $H^1(S^1 \times R^3, Z_{\tilde{2}})$. For supersymmetric models, supersymmetry transformations impose some restrictions on the twist assignments of the superfield component fields [5].

Grassmann fields cannot acquire vacuum expectation value (vev) since we require the vacuum value to be a scalar representation of the Lorentz group. Thus, the question is focused on the two scalars. The twisted scalar cannot acquire nonzero vev [4], consequently, only untwisted scalars are allowed to develop vev's.

In the literature, twisted fields have frequently been used, for example in the Scherk–Schwarz mechanism [26], where the harmonic expansion of the fields is of the form

$$\phi(x,y) = \mathrm{e}^{\mathrm{i}my} \sum_{n=-\infty}^{\infty} \phi_n(x)\,\mathrm{e}^{\frac{i2\pi ny}{L}}. \tag{1}$$

The 'm' parameter incorporates the twist mentioned above. This treatment is closely related to automorphic field theory [21, 22] in more than four dimensions (which is an alternative to the one used by us). The Scherk–Schwarz mechanism is a well-known mechanism that generates supersymmetry breaking and is frequently used for compactifications in extra dimensions models [23, 25, 28]. Also interesting generalizations of the compactification procedure using orbifolds have been introduced, in order to solve the lack of fermion chirality in odd dimensions [27, 29] (for a modern enlighting treatment see [24]). It would be worthwhile mentioning interesting alternatives to the works mentioned above such as non-commutative field theories at finite temperature [35, 38–40] and non-commutative field theories [32, 36, 37], where in some of them the gauge symmetry breaking problem is considered and the gauge Higgs unification in higher dimensions is also studied [30] (for a recent application of non-commutative field theories, see [31]).

The model under consideration is described by the global $N = 1$, $d = 4$ supersymmetric Lagrangian

$$\mathcal{L} = \left[\Phi_1^+\Phi_1\right]_D + [\Phi^+\Phi]_D + \left[\frac{m_1}{2}\Phi^2 + \frac{g_1}{6}\Phi^3 + \frac{m}{2}\Phi_1^2 + g\Phi\Phi_1^2\right]_F + \text{H.c.}, \tag{2}$$

where Φ_1, Φ are chiral superfields and the superpotential from which the interaction part of the lagrangian arises is $\left[\frac{m_1}{2}\Phi^2 + \frac{g_1}{6}\Phi^3 + \frac{m}{2}\Phi_1^2 + g\Phi\Phi_1^2\right]_F$. In the above

$$\Phi = \varphi_u(x) + \sqrt{2}\theta\psi_u(x) + \theta\theta F_{\varphi_u} + \mathrm{i}\partial_\mu\varphi_u(x)\theta\sigma^\mu\bar{\theta} \\ - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\partial_\mu\psi_u(x)\sigma^\mu\bar{\theta} - \frac{1}{4}\partial_\mu\partial^\mu\varphi_u^+(x)\theta\theta\bar{\theta}\bar{\theta} \tag{3}$$

is a left chiral superfield. It contains the untwisted scalar field components and the untwisted Weyl fermion. Although the untwisted scalar is complex, we shall use the real components which will be the representatives of the sections of the trivial bundle classified by $H^1(S^1\times R^3, Z_{\tilde{2}})$. Moreover,

$$\Phi_1 = \varphi_t(x) + \sqrt{2}\theta\psi_t(x) + \theta\theta F_{\varphi_t} + \mathrm{i}\partial_\mu\varphi_t(x)\theta\sigma^\mu\bar{\theta} \\ - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\partial_\mu\psi_t(x)\sigma^\mu\bar{\theta} - \frac{1}{4}\partial_\mu\partial^\mu\varphi_t^+(x)\theta\theta\bar{\theta}\bar{\theta} \tag{4}$$

is another left chiral superfield containing the twisted scalar field and the twisted Weyl fermion. Writing down (2) in a component form, we get (writing Weyl fermions in the Majorana representation):

$$\begin{aligned}\mathcal{L} = {} & \partial_\mu\varphi_u^+\partial^\mu\varphi_u - \left|m_1\varphi_u + \frac{g_1}{2}\varphi_u\varphi_u + g\varphi_t^2\right|^2 + \mathrm{i}\overline{\Psi}_t\gamma^\mu\partial_\mu\Psi_t - \frac{1}{2}m\overline{\Psi}_t\Psi_t \\ & - \frac{g_1}{4}(\overline{\Psi}_u\Psi_u - \overline{\Psi}_u\gamma_5\Psi_u)\varphi_u - \frac{g_1}{4}(\overline{\Psi}_u\Psi_u + \overline{\Psi}_u\gamma_5\Psi_u)\varphi_u^+ + \partial_\mu\varphi_t^+\partial^\mu\varphi_t \\ & - |m\varphi_t + 2g\varphi_t\varphi_u|^2 + \mathrm{i}\overline{\Psi}_u\gamma^\mu\partial_\mu\Psi_u - \frac{1}{2}m_1\overline{\Psi}_u\Psi_u \\ & - \frac{g}{4}(\overline{\Psi}_t\Psi_t - \overline{\Psi}_t\gamma_5\Psi_t)\varphi_u - \frac{g}{4}(\overline{\Psi}_t\Psi_t + \overline{\Psi}_t\gamma_5\Psi_t)\varphi_u^+.\end{aligned} \tag{5}$$

We can explicitly check that moebiosity is conserved at all interaction vertices, i.e. equals h_0. The moebiosity of φ_u and Ψ_u is h_0, while for φ_t and Ψ_t is h_1. One can use the Z_2 cyclic group properties to prove that the Lagrangian (5) has moebiosity h_0. The complex field φ_u can be written in terms of real components as $\varphi_u = \chi + \mathrm{i}\varphi_{u_2}/\sqrt{2}$, where $\chi = v + (\varphi_{u_1})/\sqrt{2}$ (v is the classical value). Thus, φ_{u_1} and φ_{u_2} are real untwisted field configurations belonging to the trivial element of $H^1(S^1 \times R^3, Z_{\tilde{2}})$ and satisfying periodic boundary conditions in the compactified dimension. The twisted scalar field can be written as $\varphi_t = (\varphi_{t_1} + \mathrm{i}\varphi_{t_2})/\sqrt{2}$, since, this field, being a member of the non-trivial element of $H^1(S^1 \times R^3, Z_{\tilde{2}})$ cannot acquire a vev. The tree-order masses of the two Majorana fermion fields and the four bosonic fields are calculated to be

$$\begin{aligned} m_{b_1}^2 &= m_1^2 + 3g_1m_1v + 3g_1^2v^2/2 \\ m_{b_2}^2 &= m_1^2 + g_1m_1v + g_1^2v^2/2 \\ m_{t_1}^2 &= m^2 + 4gmv + 4g^2v^2 + g^2m_1v/\sqrt{2} + g^2g_1v^2/4 \\ m_{t_2}^2 &= m^2 + 4gmv - g^2m_1v/\sqrt{2} - g^2g_1v^2/4 \\ m_{f_1} &= m_1 + g_1v, \qquad m_{f_2} = m + 2gv. \end{aligned} \tag{6}$$

In (6) m_{b_1}, m_{b_2} are the masses of the untwisted bosons (φ_{u_1} and φ_{u_2} respectively), m_{t_1}, m_{t_2} are the masses of the twisted bosons (φ_{t_1} and φ_{t_2}) and, finally, m_{f_1}, m_{f_2} are the untwisted Majorana fermion and twisted Majorana fermion masses respectively (Ψ_u and Ψ_t). We can check that the general tree level result for theories with rigid supersymmetry in terms of chiral superfields is satisfied (see [7]), i.e.,

$$STr(M^2) = \sum_j (-1)^{2j}(2j+1)m_j^2 = 0. \tag{7}$$

Also, the following relations hold true:

$$m_{b_1}^2 + m_{b_2}^2 = 2m_{f_1}^2, \qquad m_{t_1}^2 + m_{t_2}^2 = 2m_{f_2}^2. \tag{8}$$

Since twisted scalars cannot acquire vacuum expectation value, supersymmetry is not spontaneously broken at tree level, like in the O' Raifeartaigh models. The auxiliary field equations,

$$F_{\varphi_u}^{+} = m_1\varphi_u + \frac{g_1}{2}\varphi_u^2 + g\varphi_t^2 = 0 \qquad F_{\varphi_t}^{+} = m\varphi_t + 2g\varphi_u\varphi_t = 0, \tag{9}$$

imply that $\varphi_u = 0$ and $\varphi_t = 0$ and consequently $v = 0$, thus, at tree level, no spontaneous supersymmetry breaking occurs.

We now proceed by assuming that the topology is changed to $S^1 \times R^3$, while the local geometry remains Minkowski. The metric is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x_1^2 - \mathrm{d}x_2^2 - \mathrm{d}x_3^2, \tag{10}$$

with $-\infty < x_2, x_3, t < \infty$ and $0 < x_1 < L$ with the points$x_1 = 0$ and $x_1 = L$ periodically identified. The boundary conditions for the fields are

$$\begin{aligned} \varphi_u(x_1, x_2, x_3, t) &= \varphi_u(x_1 + L, x_2, x_3, t) \\ \varphi_t(x_1, x_2, x_3, t) &= -\varphi_t(x_1 + L, x_2, x_3, t) \\ \Psi_u(x_1, x_2, x_3, t) &= \Psi_u(x_1 + L, x_2, x_3, t) \\ \Psi_t(x_1, x_2, x_3, t) &= -\Psi_t(x_1 + L, x_2, x_3, t). \end{aligned} \tag{11}$$

In order to calculate the effective potential of the theory, we Wick rotate the time direction $t \to \mathrm{i}t$ thus giving the background metric the Euclidean signature [13]. The twisted fermions and twisted bosons will be summed over odd Matsubara frequencies, while the untwisted fermions and untwisted scalars will be summed over even Matsubara frequencies [2, 6]. We shall adopt the $\overline{DR}'$ renormalization scheme [7]. The Euclidean effective potential the at one-loop level is

$$\begin{aligned} V = V_0 + \frac{1}{64\pi^2 L}\sum_{n=-\infty}^{\infty}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\Bigg(\ln\left[k^2 + \frac{4\pi^2n^2}{L^2} + m_{b_1}^2\right] \\ - 2\ln\left[k^2 + \frac{4\pi^2n^2}{L^2} + m_{f_1}^2\right] + \ln\left[k^2 + \frac{4\pi^2n^2}{L^2} + m_{b_2}^2\right] \\ - 2\ln\left[k^2 + \frac{\pi^2(2n+1)^2}{L^2} + m_{f_2}^2\right] + \ln\left[k^2 + \frac{\pi^2(2n+1)^2}{L^2} + m_{t_1}^2\right] \\ + \ln\left[k^2 + \frac{\pi^2(2n+1)^2}{L^2} + m_{t_2}^2\right]\Bigg). \end{aligned} \tag{12}$$

V_0 includes the tree and the one-loop continuum corrections,

$$\begin{aligned} V_0 = m_1^2v^2 + g_1^2m_1v^3 + \frac{g_1^2v^4}{4} + \frac{1}{64\pi^2}\Bigg(m_{b_1}^4\left(\ln\left[\frac{m_{b_1}^2}{\mu^2}\right] - \frac{3}{2}\right) \\ + m_{b_2}^4\left(\ln\left[\frac{m_{b_2}^2}{\mu^2}\right] - \frac{3}{2}\right) + m_{t_1}^4\left(\ln\left[\frac{m_{t_1}^2}{\mu^2}\right] - \frac{3}{2}\right) + m_{t_2}^4\left(\ln\left[\frac{m_{t_2}^2}{\mu^2}\right] - \frac{3}{2}\right) \\ - 2m_{f_1}^4\left(\ln\left[\frac{m_{f_1}^2}{\mu^2}\right] - \frac{3}{2}\right) - 2m_{f_2}^4\left(\ln\left[\frac{m_{f_2}^2}{\mu^2}\right] - \frac{3}{2}\right)\Bigg), \end{aligned} \tag{13}$$

and μ is the renormalization scale, being of the order of the largest mass [10]. Furthermore, we shall assume that $mL \simeq 1$. This is required for the validity of perturbation theory [11, 16].

It is well known that when one considers only twisted fermions and untwisted bosons in $S^1 \times R^3$ (like in thermal field theories), vacuum contributions $\sim L^{-4}$ do not cancel and supersymmetry is spontaneously broken. The non-cancelation occurs because bosons and fermions satisfy different boundary conditions. In our model the field content is such that the cancelation of vacuum contributions is being enforced, after having included all topologically inequivalent allowed field configurations.

The leading-order contribution to the one-loop effective potential is now given by [2, 19, 20]:

$$\begin{aligned}
V = {} & m_1^2 v^2 + g_1^2 m_1 v^3 + \frac{g_1^2 v^4}{4} - \frac{3\left(2m_{f_1}^4 - m_{b_1}^4 - m_{b_2}^4\right)}{4096\pi^4} - \frac{3\left(2m_{f_2}^4 - m_{t_1}^4 - m_{t_2}^4\right)}{256\pi^4} \\
& + \frac{3\left(2m_{f_1}^4 - m_{b_1}^4 - m_{b_2}^4 + 2m_{f_2}^4 - m_{t_1}^4 - m_{t_2}^4\right)}{128\pi^2} \\
& + \frac{(\gamma - \ln[4\pi])\left(2m_{f_1}^4 - m_{b_1}^4 - m_{b_2}^4\right)}{1024\pi^4} + \frac{\left(\gamma + \ln\left[\frac{2}{\pi}\right]\right)\left(2m_{f_2}^4 - m_{t_1}^4 - m_{t_2}^4\right)}{64\pi^4} \\
& + \frac{\left(2m_{f_1}^3 - m_{b_1}^3 - m_{b_2}^3\right)}{384L\pi^3} - \frac{\left(2m_{f_1}^2 - m_{b_1}^2 - m_{b_2}^2\right)}{768\pi^2 L^2} + \frac{\left(2m_{f_2}^2 - m_{t_1}^2 - m_{t_2}^2\right)}{384\pi^2 L^2} \\
& + \frac{2m_{f_1}^4 \ln\left[Lm_{f_1}\right] - m_{b_2}^4 \ln\left[Lm_{b_2}\right] - m_{b_1}^4 \ln\left[Lm_{b_1}\right]}{1024\pi^4} \\
& + \frac{2m_{f_2}^4 \ln\left[Lm_{f_2}\right] - m_{t_2}^4 \ln\left[Lm_{t_2}\right] - m_{t_1}^4 \ln\left[Lm_{t_1}\right]}{64\pi^4} \\
& - \frac{\left(2m_{f_1}^4 \ln\left[\frac{m_{f_1}^2}{\mu^2}\right] - m_{b_2}^4 \ln\left[\frac{m_{b_2}^2}{\mu^2}\right] - m_{b_1}^4 \ln\left[\frac{m_{b_1}^2}{\mu^2}\right]\right)}{64\pi^2} \\
& - \frac{\left(2m_{f_2}^4 \ln\left[\frac{m_{f_2}^2}{\mu^2}\right] - m_{t_2}^4 \ln\left[\frac{m_{t_2}^2}{\mu^2}\right] - m_{t_1}^4 \ln\left[\frac{m_{t_1}^2}{\mu^2}\right]\right)}{64\pi^2}.
\end{aligned} \tag{14}$$

Since relation (8) holds, the terms proportional to $\frac{1}{L^2}$ cancel out [12]. Also, the minimum of the potential vanishes at $v = 0$ and supersymmetry is preserved. Indeed, upon expanding (14) for small values of v we get

$$V \simeq m_1^2 v^2 + O(v^3). \tag{15}$$

In figure 1 we plot the effective potential for the limiting case $mL = 1$. The other numerical values are chosen to be $m_1 = 200$, $m = 7000$, $g_1 = 0.001$, $g = 0.09$, $\mu = 7000$.

Next, we introduce into the Lagrangian (5) a term of the form $g_3\chi^2\varphi_{u_2}^2$, where g_3 is a dimensionless coupling constant ($\chi = v + \varphi_{u_1}/\sqrt{2}$). This term, being non-holomorphic and hard, breaks supersymmetry explicitly. Since χ develops a vev, φ_{u_2} will acquire an additional mass term of the form $g_3 v^2$. This way, the masses of the fields now become

$$\begin{aligned}
m_{b_1}^2 &= m_1^2 + 3g_1 m_1 v + 3g_1^2 v^2/2 \\
m_{b_2}^2 &= m_1^2 + g_1 m_1 v + g_1^2 v^2/2 + g_3 v^2 \\
m_{t_1}^2 &= m^2 + 4gmv + 4g^2v^2 + g^2 m_1 v/\sqrt{2} + g^2 g_1 v^2/4 \\
m_{t_2}^2 &= m^2 + 4gmv - g^2 m_1 v/\sqrt{2} - g^2 g_1 v^2/4 \\
m_{f_1} &= m_1 + g_1 v, \qquad m_{f_2} = m + 2gv.
\end{aligned} \tag{16}$$

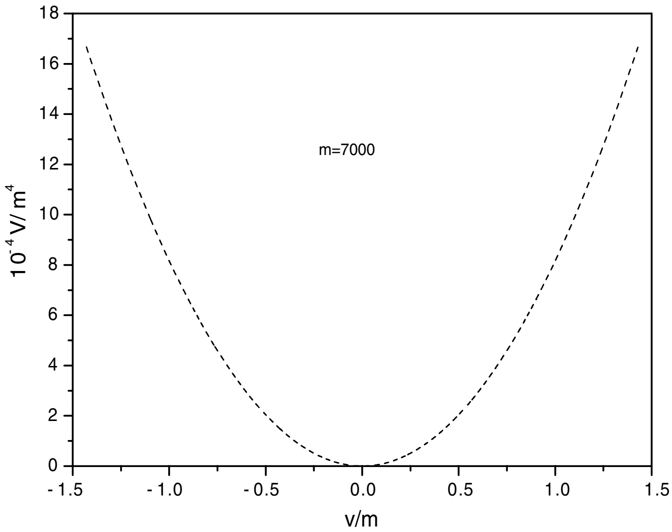

Figure 1. The supersymmetric effective potential.

As expected, supersymmetry is now broken and relation (7) becomes

$$2m_{f_1}^2 - m_{b_1}^2 - m_{b_2}^2 = g_3 v^2, \qquad m_{t_1}^2 + m_{t_2}^2 = 2m_{f_2}^2. \tag{17}$$

One can see that the supersymmetric minimum at $v = 0$ is still preserved. Indeed, V can be written as

$$V \simeq \left(m_1^2 + \frac{g_3}{768\pi^2 L^2} \right) v^2 + O(v^3). \tag{18}$$

Upon closer examination, we can see that in the continuum limit, the supersymmetric vacuum becomes metastable and a second non-supersymmetric vacuum appears. Including finite size corrections, we see that for small L the effective potential has a unique supersymmetric minimum at $v = 0$. As L increases, a second minimum develops, which becomes supersymmetric at the critical value $L_c = \frac{1}{21571}$. When $L > L_c$, the second minimum breaks supersymmetry and becomes energetically more preferable than the supersymmetric one [14, 15]. This said behavior of the potential is always valid whenever $g_3 \gg g_1$ and for $\frac{m_1}{m} \ll g_3$. Using the same numerical values as before, we plot the effective potential for $g_3 = 0.5$, first in the continuum limit (figure 2), and then including L-dependent corrections (figure 3).

Let us discuss the above results. g_3, g_1 are couplings among the untwisted superfield, g_3 corresponding to the supersymmetry breaking term. If the g_3 interaction is stronger than g_1 and if the mass (m) of the twisted superfield is larger than the untwisted one (m_1), then the following picture occurs. For a small length L of the S^1 dimension supersymmetry is not broken (figure 3). As L grows larger, a second minimum appears which is not supersymmetric $(L > L_c)$. There exists a small barrier separating the two minima (figure 3), and there is a possibility of barrier penetration between them. This resembles the first-order phase transition picture.

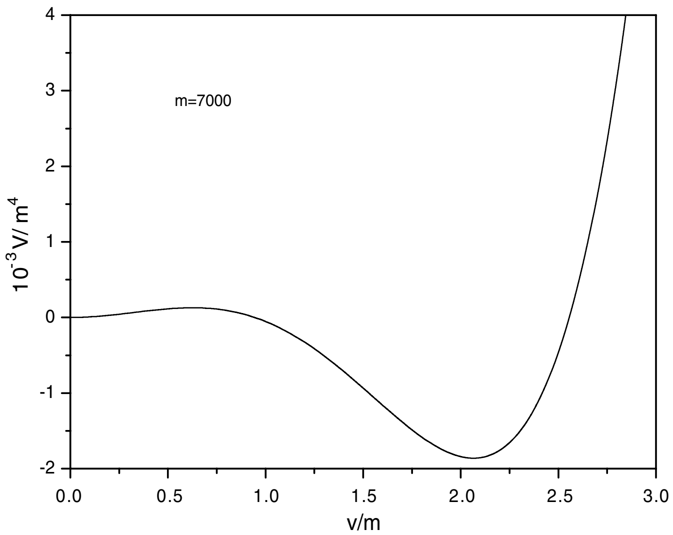

Figure 2. The continuum effective potential.

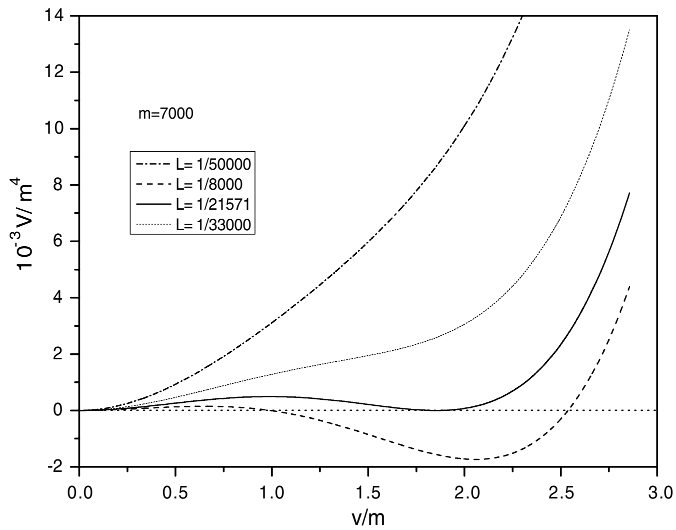

Figure 3. The effective potential including finite size corrections.

Discussion

In this section a brief qualitative application (although fictitious) of the above results is presented. We begin with a toy universe that has just come out of its strong gravity period, and its particle content (matter) is described by (5) with the addition of the hard supersymmetry breaking term $g_3\chi^2\varphi_{u_2}^2$. The back reaction of gravity on field theory is considered small (i.e. field theory calculations made in the previous part considering flat background, are consistent). This toy universe's expansion is described by

$$ds^2 = dt^2 - a^2(t)\,dx_1^2 - b^2(t)\,dx_2^2 - c^2(t)\,dx_3^2, \tag{19}$$

a homogeneous Clifford–Klein metric (known as the Bianchi I cosmological model), with x_1, x_2, x_3, as in (10). In (19), $a(t), b(t), c(t)$, describe the scale factors of the two infinite and of the compact dimension respectively. Also we assume

$$a(t) = b(t) = kc(t), \tag{20}$$

with $k \gg 1$.

Figure 3 motivates us to think as follows. At small lengths the toy universe's ground state is the supersymmetric vacuum, although we had broken supersymmetry using a hard term, something usually unexpected. As the length of the compact dimension grows, the toy universe 'acknowledges' the presence of the other vacuum (in terms of its quantum one-loop effective potential) and at some point quantum penetrates to the other vacuum, the non-supersymmetric one. Therefore, at small lengths of the compact dimension, supersymmetry was not broken and as the length grows, supersymmetry breaking occurs. This observation motivated the use of non-trivial topology in our calculations. It seems that using a compact dimension in the present model, supersymmetry breaking occurs dynamically after some critical length of the compact dimension, although supersymmetry is expected to be broken for all lengths (this would exactly be the effect of a hard supersymmetry breaking term).

Let us now do some toy cosmology on this toy universe. $V(0)$ is the minimum of the effective potential at the origin (note $V(0) = 0$), and $V(v_1)$ is the minimum after quantum penetration (the non-supersymmetric vacuum). We fix this toy universe's cosmological constant to be $(8\pi G)^{-1}\Lambda = -\Delta V = -(V(v_1) - V(0))$ (we quote the reason later on). It can easily be seen that $\Lambda > 0$.

In general, the Friedmann equation describing its evolution is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{8\pi G}\right), \tag{21}$$

referred to the x_1 dimension (the analysis on the other dimension and to the compact one is similar using (20) and for brevity we omit it. For details see [5]).

When this toy universe is at the $V(0)$ vacuum state, the energy density is $\rho = V(0) = 0$. The Friedmann equation reads

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{\Lambda}{8\pi G}\right), \tag{22}$$

and without getting into much detail (see [5]), an inflationary solution follows in all space dimensions, being of the form

$$a(t) \sim e^{\sqrt{\Lambda}t}, \tag{23}$$

with $a(t) = b(t) = kc(t)$. Note that the rate of the expansion is the same for all dimensions. As this toy universe inflates, its quantum vacuum state is the supersymmetric vacuum, until for some length quantum tunneling occurs (due to one-loop quantum effects), and the new vacuum state is $V(v_1)$, the new minimum of the effective action. During the quantum vacuum penetration, the energy release (something like latent heat) [34] is of the order L_p^{-4} which thermalizes the matter content at a temperature T_p, with

$$L_p^{-4} \sim T_p^4, \tag{24}$$

L_p and T_p characterizing the 'phase transition' point.

After thermalization, the energy density is $\rho \sim T_p^4 + V(v_1)$ and the Friedmann equation reads

$$\left(\frac{\dot{a}}{a}\right)^2 \sim \frac{8\pi G}{3}\left(T_p^4\right), \tag{25}$$

(we fixed Λ in order to cancel the value of $V(v_1)$). So after vacuum penetration the toy universe follows a radiation dominated expansion (note that the maximum temperature ever reached was the thermalization temperature T_p [34]).

The above is a very attractive picture, although fictitious. Let us point out its main features. One starts with a toy universe filled with fermions and bosons interacting in a non-supersymmetric way. The toy universe is at a supersymmetric vacuum (unexpectedly) when its magnitude (specifically the compact dimension magnitude) is small, but as it evolves spatially (inflation in our setup, or alternatively dynamical Casimir effect [18]) quantum penetrates to a non-supersymmetric vacuum, which is energetically preferable. So at the early toy universe, supersymmetry was not broken (at least the vacuum quantum state did not realize broken susy), although the matter content of it, interacts in a non-supersymmetric way, but susy breaks dynamically (quantum tunneling) [14, 15] when the toy universe evolves at larger magnitudes.

It has to be noted that if, in this toy universe, observations do not suggest non-trivial topology in its spatial dimensions, then the compact dimension magnitude must be larger than the particle horizon (which can be achieved during inflation).

Conclusions

We have investigated the possibility of altering the spacetime topology, without breaking $N = 1, d = 4$ supersymmetry. We have explicitly demonstrated, by means of a toy model, that such a construction is possible in the case of $S^1 \times R^3$ topology. Introducing an explicit supersymmetry breaking term, we find that, under suitable conditions, a second supersymmetric vacuum appears. For small length of the S^1 dimension, only the supersymmetric vacuum appears. As the length grows larger the second minimum appears which, after a critical value, becomes non-supersymmetric. This picture resembles the first-order phase transition picture.

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